



ANALYSIS OF THE EFFICIENCY OF PUBLIC EXPENDITURE ON EDUCATION IN ELEMENTARY EDUCATION IN THE STATE OF ALAGOAS

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ABSTRACT

Objective: This article analyzes the efficiency of public spending on education in the municipal public network in elementary education, in all cities in Alagoas, using the data envelopment analysis method, DEA.

Theoretical Framework: The present work used the variable return to scale (BCC) model, since we would be dealing with municipalities with varied realities. There are two basic ways for a unit that is inefficient to become efficient. The first would be to reduce resources (inputs) and keep products constant, which would be an input-oriented orientation; the second would be doing the opposite, increasing the result (products) from the same amount of inputs, an orientation focused on outputs.

Method: The data envelopment analysis method - DEA is used to estimate efficiency with traditional inputs such as school infrastructure, capital, labor and municipal wealth. As output, the Portuguese and mathematics grades in the 9th grade, in the Saeb 2019 test, are considered, multiplied by the students enrolled in this year of education; as well as the inverse of per capita expenditure on elementary education.

Results and Discussion: It was concluded that most of the benchmarkings are represented by municipalities with smaller inhabitants, indicating that the population increase in municipalities increases the demand for education.

Research Implications: Thus, based on technical efficiency analyses, public policies can be proposed that mitigate the deficiencies presented in elementary education.

Originality/Value: This study contributes to the literature by estimating the efficiency of municipalities in terms of public spending on education and the efficiency results are the grades of public school students. The research introduces a new output to measure efficiency, and points to new evidence for public educational policies.

Keywords: Education, Efficiency, Data Envelopment Analysis (DEA), Public Policies.

ANÁLISE DA EFICIÊNCIA DOS GASTOS PÚBLICOS EM EDUCAÇÃO NO ENSINO FUNDAMENTAL DO ESTADO DE ALAGOAS

RESUMO

Objetivo: Este artigo analisa a eficiência dos gastos públicos em educação na rede pública municipal no ensino fundamental, em todas as cidades de Alagoas, utilizando-se o método da análise envoltória de dados, DEA.

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Referencial Teórico: O presente trabalho utilizou-se do modelo retorno variável de escala (BCC), já que estaríamos diante de municipalidades com realidades variadas. Há duas formas básicas de uma unidade que se encontra de forma ineficiente tornar-se eficiente. A primeira seria diminuindo os recursos (insumos) e mantendo constantes os produtos, o que seria uma orientação voltada aos inputs; a segunda seria fazendo o inverso, aumentando o resultado (produtos) a partir de uma mesma quantidade de insumos, uma orientação voltada aos outputs.

Método: Utiliza-se para estimar a eficiência o método da análise envoltória de dados - DEA com inputs tradicionais como infraestrutura da escola, capital, trabalho e riqueza do município. Como output, são consideradas as notas de português e matemática na 9ª série, na prova Saeb 2019, multiplicadas pelos alunos matriculados neste ano do ensino; bem como o inverso das despesas per capita no ensino fundamental.

Resultados e Discussão: Concluiu-se que a maior parte dos benchmarkings são representados pelos municípios que possuem menores habitantes, indicando que o aumento populacional dos municípios aumenta a demanda por educação.

Implicações da Pesquisa: Assim, com base nas análises de eficiência técnica, pode-se propor políticas públicas que mitiguem as deficiências apresentados no ensino fundamental.

Originalidade/Valor: Este estudo contribui para a literatura ao estimar a eficiência dos municípios no tocante aos gastos públicos em educação e os resultados da eficiência são as notas dos alunos da rede pública. A pesquisa introduz novo output para medir eficiência, e aponta novas evidências para políticas públicas educacionais.

Palavras-chave: Educação, Eficiência, Análise Envoltória de Dados (DEA), Políticas Públicas.

ANÁLISIS DE LA EFICIENCIA DEL GASTO PÚBLICO EN EDUCACIÓN EN EDUCACIÓN PRIMARIA EN EL ESTADO DE ALAGOAS

RESUMEN

Objetivo: Este artículo analiza la eficiencia del gasto público en educación en la red pública municipal en educación básica, en todas las ciudades de Alagoas, utilizando el método de análisis envolvente de datos, DEA.

Marco Teórico: El presente trabajo utilizó el modelo de retorno variable a escala (BCC), ya que estaríamos tratando con municipios con realidades variadas. Hay dos formas básicas para que una unidad que es ineficiente se vuelva eficiente. La primera sería reducir los recursos (insumos) y mantener constantes los productos, lo que sería una orientación orientada a los insumos; el segundo sería hacer lo contrario, aumentar el resultado (productos) a partir de la misma cantidad de insumos, una orientación centrada en los resultados.

Método: El método de análisis envolvente de datos - DEA se utiliza para estimar la eficiencia con insumos tradicionales como infraestructura escolar, capital, mano de obra y riqueza municipal. Como resultado, se consideran las calificaciones de portugués y matemáticas del 9º grado, en la prueba Saeb 2019, multiplicadas por los estudiantes matriculados en este año de educación; así como la inversa del gasto per cápita en educación primaria.

Resultados y Discusión: Se concluyó que la mayoría de los benchmarkings están representados por municipios de menor población, lo que indica que el aumento poblacional en los municipios aumenta la demanda de educación.

Implicaciones de la Investigación: Así, a partir de análisis de eficiencia técnica se pueden proponer políticas públicas que mitiguen las deficiencias presentadas en la educación básica.

Originalidad/Valor: Este estudio contribuye a la literatura al estimar la eficiencia de los municipios en términos de gasto público en educación y los resultados de eficiencia son las calificaciones de los estudiantes de las escuelas públicas. La investigación introduce un nuevo resultado para medir la eficiencia y apunta a nueva evidencia para las políticas educativas públicas.

Palabras clave: Educación, Eficiencia, Análisis Envoltorio de Datos (DEA), Políticas Públicas.



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1 INTRODUCTION

International experiences demonstrate that investments in education can boost a country's economy, making it more productive and also providing non-economic benefits, such as increased life expectancy, reduced crime and increased quality of life in general. (ALMEIDA; GASPARINI, 2011).

The promulgation of the Federal Constitution of 1988, together with the approval of changes in legislation in recent decades, guaranteed greater access to education in Brazil, notably the extension of the duration of free and compulsory primary education in public schools (BRASIL, 1988). According to data provided by the National Institute of Education and Research (INEP), Brazil has been increasing total public investments in education in relation to gross domestic product (GDP). Between 2010 and 2015 there was an increase from 5.6% to 6.2% in relation to GDP.

However, when relating the legislation that determined the responsibilities of federative entities in relation to expanding the provision of basic education, it is noted that the majority of these responsibilities fell on states and municipalities, some even mandatory, without due proportional increase. of financial resources directed to such purposes (CERQUEIRA, 2018). Given this, the responsibility of ensuring elementary education for school-age children, in addition to other duties, such as increasing the provision of early childhood education, was more than challenging for most municipalities, given the low economic activity found in part considerable amount of them. According to IBGE data, of the 5,570 Brazilian municipalities, 49.9% have public administration as their main economic activity. Of these, 70% are in rural areas and 96% are small (CERQUEIRA, 2018) (IBGE, 2019).

The results of all these factors highlighted by some education quality indicators are not encouraging when compared to other countries. Of the 70 countries evaluated in PISA in 2015, Brazil ranked 63rd in science, 59th in reading, and 66th in mathematics; in other words, despite the increase in spending in this sector, no increase in the effective quality of education was noted (BRASIL, 2018; PISA, 2015), in 2022 Brazil drops its position to 65th. Internal methods that monitor the quality of teaching, such as the IDEB (Basic Education Development Index), created in 2007, which combines the results of two concepts in a single indicator: the school



flow and the average performance in assessments such as the Prova Brazil at the municipal level has also pointed out that Brazilian students in public schools are well below the levels considered reasonable for a satisfactory education.

The Brazilian state of Alagoas would, theoretically, benefit the most from an increase in the efficiency of investments in education, as it has the lowest HDI (Human Development Index) in Brazil, that is, it is a region with greater development needs. Furthermore, because this development index is composed of data such as life expectancy, education and GDP per capita, this region could be exponentially leveraged with better quality in education, as this would be reflected in the two other factors that form the HDI.

Thus, considering that resources are scarce and increasing taxes to subsidize more spending would be unfeasible, better quality management of resources allocated to education could provide significant economic growth for the State, as it has a population with a low average level of education. and with internal disparities, which could be combatted with the formation of more qualified human capital. Furthermore, there are studies that demonstrate a significant inefficiency in the use of these public resources, with room for optimization (SAVIAN; BEZERRA, 2013; DINIZ, 2012).

Elementary education was the stage of the educational system evaluated due to the fact that, in addition to being universal, its importance is clear in the formation of citizens, both educational and ethical. This phase, as several studies demonstrate, has direct impacts on undergraduate, postgraduate studies and, consequently, on the human capital of a country (DIAZ, 2012). Therefore, this article aims to assess the efficiency and quality of public spending on education in primary education in the municipal education network in cities in the state of Alagoas, with the purpose of verifying and evaluating which cities are most and least efficient in providing of this educational service and, based on its conclusions, carry out public educational policies that are more in line with the needs and reality presented in that state.

2 THEORETICAL FRAMEWORK

Education is one of the most determining factors for social mobility, enabling the advancement of those individuals who are in lower social classes and reducing differences in opportunities in the job market. Furthermore, several studies show the direct relationship between the level of education and salary increases, leading to economic development for the country (WILBERT; D'ABREU, 2013). Still according to the author, education is not a pure public good, its consumption by one person prevents its enjoyment by another; that is, it is



exclusionary. In this case, the private sector is not capable of achieving the economic objective of allocating resources and services where there is a need, such as in regions where people do not have the financial means to cover the costs of education. The State then acts to correct this market failure, treating this sector through public policies, aiming for a more comprehensive offer of the service and allocating it in a way that reduces levels of inequality and promotes greater social mobility. For this, taxes paid by contributing citizens are used, consequently generating the duty of governments to use these resources in the most efficient way.

In Brazil, education is predominantly public, around 80% of primary education comes from federal, state and municipal schools, and around 70% in the case of secondary education, with this share decreasing due to school dropouts at this stage. (DELGADO, 2008) However, based on literature from recent decades, the scarcity of financial resources and their management are identified as the biggest problems with the inefficiency of public policies in the area of education, (DINIZ; CORRAR, 2011).

With a focus on the management and efficiency of public spending, the Magna Carta restructured Brazilian federalism, especially with regard to the distribution of public revenues (VARELA, 2008). In this context, according to Guerreiro (2006), taking into account Brazil's regional disparities and the Union's challenges to reduce imbalances between local revenue capacity and the provision of public services, the Federal Constitution increased the distribution of federal funds to states and municipalities through the State Participation Fund (FPE) and Municipal Participation Fund (FPM).

It is worth highlighting the evaluation of public policies in the area of education, regarding the actions that the state takes to favor the demands of some social groups, sometimes with negative impacts on others. A more objective evaluation of these actions and an analysis of their results would make it possible to correct some mistakes in public spending (SILVA et al., 2015).

The evaluation of public policies and government programs is considered essential for improving decision-making by the Public Administration; its results allow budgets to be redirected, strategies to be redefined and impacts to be measured. From 1990 onwards, the topic began to have great importance in academic research carried out in Brazil and the rest of the world, given the reduction in public budgets and, consequently, the need for greater efficiency and investments in government spending (GUINA; PHILIPPEN JUNIOR; PASSADOR, 2014).

Studies evaluating these policies generally differentiate between efficiency, effectiveness and effectiveness. Efficiency would be the relationship between financial, material and human resources with the activities/results achieved, that is, the good use of these



inputs; effectiveness concerns the ability to do something, to meet the established goal; and finally effectiveness, the observation of the incorporation of changes generated by a given program in the reality of the target population (MINAYO, 2009).

The efficiency of a social policy is measured by the relationship between the results of a program, what benefits it brought, and its expenses, what was used in its process. Efficiency assessment would then present itself as a search for restructuring the action, in order to carry it out aiming for a more satisfactory result at a lower cost (FAGUNDES; MOURA, 2009). Another reason that can be cited for the relevance of this type of assessment, as well pointed out by Savian and Bezerra (2013), is that inefficient or inappropriate public policies in the area of education could result in an increase in education without corresponding human capital. , this is the fundamental element for the social and economic development of a country.

Efficiency, in the case of education, would be measured by quality criteria; also considering that the results of investments in this area are long-term. Spending on education should result in balance in education systems and improvements in student learning levels (BRUNET, BERTÊ; BORGES, 2008). The biggest challenge in this type of assessment is defining a metric that can combine some variables into a single indicator that can reflect efficiency in public management. The core of measuring efficiency would be the definition of this standard, in order to compare various actions and check whether they are being efficient or not (DINIZ; MACEDO; CORRAR, 2012).

Thus, data envelopment analysis (DEA), since it began to be used, has been applied to evaluate efficiency in education, seeking to improve the result/cost relationship, that is, maximize results (outputs), and minimize costs (input). It began with Charnes, Cooper and Rhodes (1978), evaluating the efficiency of educational programs in Texas schools (WILBERT; D' ABREL, 2013).

As for studies related to school performance, in Brazil, the study on quality in education was developed with Saeb in 1995, late, but with adequate method and technique. These researches reflected what international work demonstrated, that the school would play a fundamental role in the students' school lives, even controlling the students' socioeconomic level. There was also a discrepancy between schools in the country compared to other countries, especially when considering public and private schools (ALVES; FRANCO, 2008)

Silva et al (2015), using the Data Envelopment Analysis (DEA) method, evaluated the efficiency in the management of elementary education resources in the cities of São Paulo, Rio de Janeiro and Belo Horizonte, in order to analyze whether changes were made or remained in social inertia during the IDEB publication period, between 2005 and 2011. They concluded that



the city of São Paulo was in social inertia in 2011, taking into account both the years of decision-making, and compared to the three municipalities in 2011. Rio de Janeiro and Belo Horizonte showed efficiency both in relation to the years and compared to other cities.

Diniz (2012), sought, through the analysis of efficiency in the management of public spending on education, to validate the thesis that conditional transfers to elementary education are detrimental to the efficiency of the application of resources. He concluded that, in general, the thesis was confirmed; that the most efficient municipalities were those with the highest cost per student, the highest number of students enrolled in basic education and that financial autonomy is positive for the efficiency in the application of these resources.

Savian and Bezerra (2013) evaluated the municipalities of Paraná regarding the efficiency of spending on education in the initial grades of elementary school, in the years 2005 and 2009, through the DEA, comparing both municipalities and mesoregions. In the end, they found an inefficiency in most municipalities, and the need to review the allocation of these resources.

3 METHODOLOGY

According to Casado and Souza (2007), Data Envelopment Analysis (DEA) is a non-parametric technique that uses mathematical programming, created to measure the efficiency of organizations that do not aim for profit, or for which there are no pre-prices. -fixed for products/inputs. This method is widely used in economic sciences, and was initially developed by Farrel (1957) and improved by Aigner et al. (1968), Forsund et. al (1977) and Färe Grosskopf and Lovell (1994) and others.

The application of the DEA consists of three phases: firstly, the DMUs (Decision Making Units) to be analyzed are defined, which in this case would be the municipalities; subsequently, the input and product variables are defined to evaluate the efficiency of the DMUs and, finally, the DEA model is applied, opting for a greater or lesser level of sophistication (LINS; MEZA, 2000) .

The DEA is based on real data, however the efficiency estimated by the methodology is relative. Using an empirical model, it points out the efficiency scores for each production unit, in order to compare them as a homogeneous set. From the analysis, an efficiency frontier is identified and each DMU evaluated will have an efficiency score between 0% and 100%. Therefore, this methodology is considered the most suitable for measuring the efficiency of the allocation of public resources, taking into account different services, identifying the



performance of the units and comparing them (SILVA et al., 2012). Santos, Freitas & Flach (2018) state that the DEA model has two classic modes, namely: CRS (Constant Returns to Scale), in the case of a constant return to scale and VRS (Variable Returns to Scale), in the case of situations in which there is variation in scale. We can point out the axiom of proportionality as the main difference between these approaches. In the case of CRS, variation in inputs results in proportional variation in products, which does not happen in the VRS model.

Furthermore, in both models, guidance can be carried out focused on inputs, seeking to measure the quantity of these variables that can be reduced while maintaining the products; or for products, we seek to maximize results (products), considering the same inputs (TSCHAFFON, MEZA, 2014). DEA-CCR has the characteristic of reducing multiple inputs and products to a single virtual input and product, calculated for each DMU. In a DMU, this relationship between input and virtual product provides us with the efficiency measure that is a function of the multipliers, which enables an objective assessment of global efficiency.

This model allows efficiency to be measured by maximizing the sum of weighted products due to the sum of weighted inputs, with the result being less than or equal (\leq) to 1, according to Charnes et al. (1978):

$$E = \frac{\text{Produto } 1 + \text{Produto } 2 + \dots + \text{Produto } n}{\text{Insumo } 1 + \text{Insumo } 2 + \dots + \text{Insumo } n} \text{ ou } E = \frac{\sum_i u_i Y_i}{\sum_j v_j X_j} \leq 1 \quad (1)$$

where Y_i is the product i , X_j is the input, j and i and v_j are the respective weights. Scores are then considered efficient when equal to 1, and inefficient when they have a lower value. Thus, it is allowed to set up a curve where those DMUs that are above it (border) are efficient, and those located below are inefficient. In relation to the BCC model, also known as VRS – Variable Returns to Scale, it assumes that DMUs assume variable returns to scale. It replaces the axiom of proportionality between inputs and outputs with the axiom of convexity. This model allows the projection of each inefficient DMU onto the efficiency frontier (envelope) established according to the different sizes of DMUs (MELLO, et al., 2008; TSCHAFFON, MEZA, 2014).

The DEA BCC determines a VRS frontier, based on increasing or decreasing returns to scale on the efficient frontier, where an increase in input may reflect an increase or decrease in output, not necessarily proportional (GUERREIRO, 2006). The present work used the variable return to scale (BCC) model, since we would be dealing with municipalities with varied realities. There are two basic ways for a unit that is inefficient to become efficient. The first



would be to reduce resources (inputs) and keep products constant, which would be an input-oriented orientation; the second would be doing the opposite, increasing the result (products) from the same amount of inputs, an orientation focused on outputs (MELLO et al., 2005).

Regarding the sample and analysis method, the research aims to analyze the efficiency and quality of per capita investments in education in the municipal elementary school network in cities in the state of Alagoas. This type of analysis has been the subject of several studies nationally and abroad, however there is no consensus on the choice of inputs and products to be used to measure efficiency (SILVA FILHO et al., 2016). In this way, the choice of variables to be used in this work were extracted based on specialized literature, more specifically on the model used by Almeida and Gasparini(2011), which seeks to analyze the qualitative aspect of spending on Education, taking into account both the level of learning in terms of expenses spent, also paying attention to the socioeconomic situation in which the student is inserted, a factor often considered significant in the final learning result.

Thus, the present study sought to collect some types of data related to traditional inputs that indicate school infrastructure, capital, work and wealth of the municipality. They are: the number of teachers in the second phase of primary education (5th to 9th grades); the average daily class hours at this stage of elementary school, multiplied by the number of students enrolled in this period; number of public elementary schools with access to the electricity network and/or that have a sewage system; Gross Domestic Product (municipal GDP per capita.

In order to capture the efficiency of public spending on education and relate it to the quality of teaching, the grades in Portuguese and mathematics in the 9th grade, in the Saeb test, multiplied by the students enrolled in this year of education will be used as a proxy for results; as well as the inverse of per capita expenditure on elementary education (criticized by international organizations for being considered low in Brazil). In this case, the opposite was applied because DEA predicts the maximization of output variables.

As input, we first selected those related to the school, structural environment, which demonstrated material and human aspects at the service of education and the student. For this, the number of teachers from the second part of primary education, professionals directly involved in teaching, were used; average class hours multiplied by the number of students enrolled in this stage of education (5th to 9th grades), which would represent the amount of education offered; and the number of municipal public schools with electricity and/or sewage systems, representing a basic structure to be used by these locations. These variables can be found in the work carried out by Almeida and Gasparini (2011); Silva, Souza and Araújo (2013) and Matos (2018).



For outputs, it was used as a proxy for the result to be achieved. In other words, it is expected that the higher the average grades in Portuguese and mathematics, multiplied by the number of students enrolled, the more the objective of education, of learning, is being exercised. This factor, together with the inverse of per capital expenditure per student, would demonstrate the efficiency index achieved in the application of resources by municipal public administrators. This variable was also used in the works of D'Abreu and Wilbert (2013), Almeida and Gasparini (2011), Soares et al., 2020, among others.

In order to measure the supply of municipal education services, the variables calculated on average were multiplied by the number of students enrolled in the 9th grade, thus seeking a proxy for the total number of students enrolled in this stage of education. The last year of elementary school was chosen as a way to evaluate this entire phase. It is worth mentioning that as of 2019, the Brazil test began to be called the Basic Education Assessment System (Saeb), a system that has existed since the 90s and which now names the set of Basic Education assessments. A large-scale assessment, developed by the National Institute of States and Educational Research Anísio Teixeira (INEP/MEC), with the objective of analyzing the quality of teaching offered by the Brazilian educational system. This test is applied to all students in the public education network, in the 4th and 9th grades of elementary school, and their averages support the calculation of the Basic Education Development Index (IDEB).

Expenses in the education function – municipal elementary education were collected on the SICONFI website - Brazilian Public Sector Accounting and Fiscal Information System), linked to the National Treasury Secretariat, divided by the number of students enrolled in elementary education to arrive at a value per capita. This analysis was carried out with the help of the free tool called SAGEPE (Data Envelopment Analysis Software). The data used were collected through the websites of the National Treasury Secretariat (STN), the Brazilian Institute of Geography and Statistics (IBGE) and the Educational Statistics System (school census microdata) of the National Institute of Educational Studies and Research Anísio Teixeira (Inep).

In relation to the municipalities of Penedo, Atalaia, Lagoa da Canoa, Barra do Santo, Passo de Camaragibe, Carneiros, Japaratinga and Santa Luzia do Norte, no data was found relating to expenditure paid in the education function – primary education for the year 2019. Thus, the moving average for the years 2015 and 2017 of these cities was used, with regard to the variable in question. Emphasizing once again that the DEA methodology is sensitive to extreme values considered atypical, these municipalities were excluded from the definition of the technological frontier, in order to prevent possible distortions, considering that these cities



have characteristics that are very different from the state average. The municipality of Junqueiro was also excluded from the analysis, given that no data was found on expenses paid in the education function – primary education in the years 2019, 2017 and 2015.

4 RESULTS AND DISCUSSIONS

Initially, it is worth highlighting the ranking of the analyzed units. Using the model, the efficiency of the DMUs was calculated, the standard efficiency frontier was constructed and those units that are taken as benchmarks of best practices were verified. Then, using the SAGEPE software, seeking to eliminate units considered false efficient and taking into account the benevolence with the DMU's in their standard efficiency, the inverted, composite and normalized composite efficiencies were used, using the variable return output-oriented scale, in the form of a ranking of municipalities.

Analyzing simultaneously the 102 municipalities in the state of Alagoas, with the exception of Maceió, Arapiraca and Junqueiro, it is observed that 19 municipalities achieved their standard efficiency scores equal to one (Standard Eff. = 1, or 100%), as shown in table 2, above. The other municipalities in this sample space are considered inefficient, due to their score being less than one. From the calculated indicator, it was found that, of the 99 municipalities that were considered for the construction of the efficiency frontier, around 20% were considered as references for the others, representing the best practices analyzed. These municipalities are those that managed to generate, given the inputs used, the best comparative result with the other cities assessed in the sample.

There were 19 municipalities that made up the efficiency frontier, considered benchmarks for inefficient municipalities: Jequiá da Praia, Murici, Poço das Trincheiras, Paripueira, Senador Rui Palmeira, Estrela de Alagoas, Carneiros, Palestine, Água Branca, Jaramataia, Campestre, Feliz Desert, Marechal Deodoro, Flexeiras, Campo Alegre, Minador do Negrão, Coruripe, Dois Riachos, Pindoba. From the point of view of educational policies, the municipalities that form the border could be taken as parameters for other locations of similar dimensions and proportions in terms of the use of inputs and the allocation of public resources. The inverted frontier coefficient, which would be the inversion of outputs and inputs; that is, what is input would become output and vice versa, together with the composite efficiency and the normalized composite efficiency, it can be used to rank the DMU's, a means of checking which is actually the most efficient among the others.



$$Efic. Composta = \frac{(Efic.padr\tilde{a}o)-(efic.invertida)+1}{2} \quad (2)$$

$$Efic. Composta normalizada = \frac{Efici\tilde{e}ncia Composta i}{maior\ valor\ efic.composta} \quad (3)$$

Therefore, observing the table above and the normalized composite efficiency, the municipality of Jequi da Praia would be the most efficient among the cities in Alagoas analyzed, using the output-oriented variable returns to scale method. This municipality has the second highest average in Portuguese (291) and the best average in mathematics (343) in the 9th year in the SAEB tests, in addition to the lowest expenditure paid in education/elementary education per capita.

The municipalities considered as outliers, Macei and Arapiraca, atypical in the sample due to their large population, were excluded from the analysis as they could create bias in the result and could not be considered to determine the efficiency frontier, however, the index for them was also calculated, especially because it is interesting to observe whether or not some of the main cities in the state, despite having atypical data, managed to achieve the reference result.

The result was very different for the two cities. The municipality of Arapiraca, with approximately 232 thousand inhabitants, achieved the reference result in educational performance, with a standard efficiency equal to 1. However, Macei, the state capital, with approximately 1 million inhabitants, obtained a standard efficiency equal to 0.48 compared with all cities in the state, using the variable return to scale method. Analyzing the inputs applied by the city of Macei, there is a much higher quantity compared to all others, which is understandable due to its size. Despite this, the result generated is not presented in the same proportion, which could explain the low standard efficiency score. Benchmarking analysis is used to identify which DMUS or DMUS are efficient and, based on this, how they can be considered references for inefficient DMUs. The DMU with the greatest overall weight is the one that should serve as a reference for the inefficient DMU to use its practices as a basis to become an efficient DMU. In other words, the efficient DMU with the highest overall weight is the reference that the inefficient DMU can visit to observe how it can improve its production practices. The table below shows which cities are considered benchmarks, and the number of municipalities that can use it as a reference to become more efficient.

**Table 1**

Efficient municipalities in the application of resources in education in the state of Alagoas in 2019 (Benchmarking)

Municipalities with Efficiency = 1 and (Benchmark)			
Campo Alegre (76)	Flexors (14)	Senator Rui Palmeira (5)	Countryside (0)
Jequiá da Praia (75)	Marshal Deodoro (14)	White Water (4)	Two Creeks (0)
Palestine (44)	Coruripe (12)	Happy Desert (3)	Star of Alagoas (0)
Pindoba (27)	Pit of the Trenches (12)	Jaramataia (2)	Negrão Minador (0)
Paripueira (20)	Rams (11)	Murici (1)	

It is observed that the cities of Campo Alegre and Jequiá da Praia are the ones that appear most often as a benchmark for the others, that is, the municipality of Campo Alegre serves as a reference for 76 cities in Alagoas, followed by Jequiá da Praia as a reference parameter. comparison for 75 municipalities in the state. The municipality of Jequiá da Praia, as previously verified, taking into account the normalized composite efficiency, would be the most efficient among the 99 cities analyzed in Alagoas, using the output-oriented variable returns to scale method.

The municipality of Murici presents itself as a benchmark and best reference for the municipality of Limoeiro de Anadia. Despite the DMUs Campestre, Dois Riachos, Estrela de Alagoas and Minador do Negrão reaching the efficiency frontier, they were not considered models for any of the 80 inefficient units. For the analysis of partners of excellence, the dominant efficient DMU constitutes a reference partner for an inefficient DMU, with two aspects being relevant to this analysis, according to Ferreira and Gomes (2009, p. 311): a) The greater the positive value of λ_k , the more important the efficient DMU is as an excellent partner, b) The more times an efficient DMU is identified as an excellent partner for inefficient DMUs, the greater its importance among the efficient DMUs. Thus, as an example, the municipality of Pindoba appears as a reference for 27 other municipalities (Decision-Making Units), in addition to itself, and appears as the best partner of excellence for the municipality of Roteiro. This means that the municipality of Roteiro can rely on the same management practices in the application of basic education resources that the municipality of Pindoba adopts to reach the efficiency frontier of the DEA model.

In order to deepen the classification of the efficiencies of the municipalities analyzed here, the classification of Barros (2017) was adopted, an adaptation of Cavalcante (2011). This option resulted from the separation into groups according to the degree of efficiency, in



municipalities considered to have high (100%), medium (80% < 99.9%), low (50% < 80%) and very low (< 50%). Thus, according to this classification, 19 cities were considered high efficiency; 30 average efficiency; 45 low-efficiency municipalities; and 5 of very low efficiency, according to table 2.

Table 2

Proportion of residents and municipalities by efficiency band

Municipalities by range of efficiency	Population put range of efficiency	Percentage of population of the 9 municipalities	Quantity of Municipalities by range of efficiency	Percentage of municipalities by range of efficiency
100%	349,200	16.93	19	19.19
99,9% - 80%	600.656	29,13	30	30,30
80% - 50%	905.675	43,92	45	45,45
< 50%	104.354	5,06	5	5,05

It is observed based on these data, obtained with the DEA analysis, that only 19.19% of elementary school students in the state benefit from efficient public services in education and 80.8% of municipalities in Alagoas underutilize their financial resources in this area; that the majority of the population and municipalities are in the low efficiency subdivision between the percentages of 80% and 50%, with 43.92% of the population and 45 cities; and that the division presented with the lowest population percentage is in the range less than 50%, very low, with 5.06% of the population and 5 cities.

It can be seen in Table 3, which lists the cities considered efficient with the group/quartile based on the population criterion, that most of those with a standard efficiency of 100% are from the groups/quartiles represented by the municipalities that have the smallest number of inhabitants, 13 municipalities out of a total of 19 are in quartiles 3 and 4, corroborating the thesis that the population increase in municipalities means an increase in demand for education, meaning that municipalities with a higher population density may have difficulties in managing the public resources allocated to teaching (SILVA, ALMEIDA, 2012).



Table 3

Relationship: Efficient municipalities X quartiles (population criterion)

MUNICIPALITIES Eff. = 1.00	BENCHMARK	GROUP/ QUARTILE	OUTPUTS		
			Average_9_LP x Mat.9year	Average_9_MT	Inv.Des.Ens.Fund. Per capita
Campo Alegre	76	1	146754	165558.6	0.0001034
Coruripe	12	1	210238.74	228364.5	0.0001431
Marshal Deodoro	14	1	160351.1	160773.2	0.0001482
Murici	1	1	115280.4	118024.2	0.0001800
White Water	4	2	55336.96	56107.52	0.0001508
Star of Alagoas	0	2	45252	45532	0.0001364
Rams	11	3	34638	35042.34	0.0000923
Two Creeks	0	3	34620.2	35569.95	0.0001045
Flexors	14	3	50220	49703.76	0.0000727
Jequiá da Praia	75	3	36667.26	43195.32	0.0086394
Paripueira	20	3	52431.84	51121.68	0.0001368
Pit of the Trenches	12	3	52070.5	53861.19	0.0001558
Senator Rui Palmeira	5	3	55932.35	57622	0.0001522
Countryside	0	4	19031.32	19124.49	0.0001410
Happy Desert	3	4	14947.65	15754.77	0.0001505
Jaramataia	2	4	15168.92	15761.02	0.0001644
Negrão Minador	1	4	17193.4	17312.4	0.0001087
Palestine	44	4	19050.06	19118.79	0.0001370
Pindoba	27	4	8172.78	8226.57	0.0001370

5 CONCLUSION

The measurement of the relative efficiency index of municipal education, more specifically of elementary education, was carried out in 99 of the 102 municipalities in Alagoas, excluding the municipalities of Maceió and Arapiraca from the examination, considering that they have a population greater than 100 thousand inhabitants, with socioeconomic indicators that are much higher than the others, as well as the municipality of Junqueiro (no data was found on expenses paid in the education function – primary education in the years 2019, 2017 and 2015). The average standard efficiency of the municipalities of Alagoas (except Maceió, Arapiraca and Junqueiro) was 78.79%, when using the output-oriented variable return to scale.

Based on the analyzes carried out in relation to all municipalities, using output-oriented variable return to scale, it was observed that only 19.19% of elementary school students in the state benefit from efficient public services in education and 80.8% of municipalities in Alagoas underutilize their financial resources in this area; that the majority of the population and municipalities are in the low efficiency subdivision between the percentages of 80% and 50%, with 43.92% of the population and 45 cities; and that the division presented with the lowest population percentage is in the range less than 50%, very low, with 5.06% of the population and 5 cities.



Taking into account the normalized composite efficiency, the municipality of Jequiá da Praia would be the most efficient among the cities in Alagoas analyzed, only 100%. This municipality has the second highest average in Portuguese (291) and the best average in mathematics (343) in the 9th year in the SAEB tests, in addition to the lowest expenditure paid in education/elementary education per capita. At the opposite extreme, with efficiency considered very low, we have the municipality of Jundiá, with a standard efficiency score of 0.412, that is, given the inputs used, the municipality generated around 60% below what would be possible if it used the best practices employed by the units that make up the border.

When relating the cities considered efficient with the group/quartile based on population criteria, it is observed that most of the so-called benchmarking are from the groups/quartiles represented by the municipalities that have the lowest number of inhabitants, 13 municipalities out of a total of 19 are of quartiles 3 and 4, corroborating the thesis that the population increase in municipalities means an increase in demand for education and greater challenges in this sector.

The DEA method was also used to point out the benchmarks of municipalities considered inefficient, also presenting those that had the highest and lowest weight. Thus, of the 19 efficient units, 15 were identified as benchmarks for inefficient DMUs, excluding the Campestre, Dois Riachos, Estrela de Alagoas and Minador do Negrão units, which were not considered models for any of the 80 inefficient units.

Thus, taking the research results as a basis, it is concluded that the DEA method made it possible to meet the research objectives, presenting a vision of the efficiency of municipal education in Alagoas, more precisely its elementary education. For future work on the topic, it is recommended that this analysis be replicated with updated data, in order to carry out a comparative study and improve elementary education management practices in municipalities in Alagoas, considering that this type of analysis allows for each unit to visualize its strengths and weaknesses, as well as its opportunities for improvement, based on its benchmark practices.

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